שאלה 1, סעיף א

1

$$3x^2(1+\ln(y))\,dx-\left(2y-\frac{x^3}{y}\right)dy=0$$

$$3x^2(1+\ln y)\,dx+\left(-2y+\frac{x^3}{y}\right)dy=0$$

$$M_y: 3x^2\cdot\frac{1}{y} \quad\Big|\quad N_x: 3x^2\cdot\frac{1}{y}$$ מדויקת ✓

$$F(x,y)=C$$

$$F=\int M\,dx=\int 3x^2\,dx+\int 3x^2\ln y\,dx=\frac{3x^3}{3}+\ln y\cdot\frac{3x^3}{3}+h(y)=x^3(1+\ln y)+h(y)$$

$$F_y=N: x^3\left(\frac{1}{y}\right)+h'(y)=-2y+\frac{x^3}{y}$$

$$F_y=\cancel{\frac{x^3}{y}}+h'(y)=-2y+\cancel{\frac{x^3}{y}}$$

$$h'(y)=-2y \rightarrow h(y)=-\frac{\cancel{2}y^2}{\cancel{2}}=-y^2$$

$$\Rightarrow F=x^3(1+\ln|y|)-y^2=C \,//$$

שאלה 1, סעיף ב

$$y'' - 3y' + 2y = \sin x + e^{2x}$$

y_H: $k^2 - 3k + 2 = 0 \qquad k_1 = 2,\ k_2 = 1$

$$y_H = C_1 \cdot e^{2x} + C_2 e^{x}$$

y_p: $\sin x,\ \cos x,\ e^{2x}$

$$y_p = A\sin x + B\cos x + xe^{2x} \cdot C$$

$$y' = A\cos x - B\sin x + \overset{C}{e^{2x}} + \overset{C}{2xe^{2x}}$$

$$y'' = -A\sin x - B\cos x + \underbrace{e^{2x} \cdot 2 + \overset{C}{2e^{2x}}}_{4e^{2x}} + \underbrace{\overset{\cdot C}{2xe^{2x} \cdot 2 \cdot C}}_{4xe^{2x}}$$

$$-A\sin x - B\cos x + \overset{C}{4e^{2x}} + \overset{C}{4xe^{2x}} - 3(A\cos x - B\sin x + \overset{C}{e^{2x}} + \overset{C}{2xe^{2x}})$$
$$+ 2(A\sin x + B\cos x + \overset{C}{xe^{2x}}) = \sin x + e^{2x}$$

$\sin x$: $1 = -A + 3B + 2A \Rightarrow 1 = A + 3B$

e^{2x}: $1 = 4C - 3C \Rightarrow \boxed{C = 1}$

$\cos x$: $0 = -B - 3A + 2B \Rightarrow 3A = B$

$\Rightarrow 1 = A + 3(3A)$

$1 = A + 9A = 10A$

$10A = 1 \Rightarrow \boxed{A = \frac{1}{10}}$

$\boxed{B = \frac{3}{10}}$

$$y_p = \frac{1}{10} \cdot \sin x + \frac{3}{10}\cos x + xe^{2x}$$

$$y(x,y) = C_1 \cdot e^{2x} + C_2 e^{x} + \frac{1}{10}\sin x + \frac{3}{10}\cos x + xe^{2x} \quad //$$

שאלה 2 סעיף א

ב

א

$$\frac{\partial u}{\partial t} = 6\frac{\partial^2 u}{\partial x^2}$$

$$u(x,t) = X(x)\cdot T(t)$$

$$\left.\begin{aligned} \frac{\partial^2 u}{\partial x^2} &= X''(x)\cdot T(t) \\ \frac{\partial u}{\partial t} &= X(x)\cdot T'(t) \end{aligned}\right\} \quad \begin{aligned} X\cdot T' &= 6\cdot X''\cdot T \\ \frac{X''}{X} &= \frac{1}{6}\frac{T'}{T} = -\lambda^2 \end{aligned}$$

$$\frac{\frac{dT}{dt}}{T} = -6\lambda^2 \rightarrow \frac{dT}{dt} = -6\lambda^2 T \Rightarrow \int\frac{dT}{T} = \int -6\lambda^2\, dt$$

$$\rightarrow \ln|T| = -6\lambda^2 t + \ln|c_1|$$

$$\ln\left(\frac{T}{c_1}\right) = -6\lambda^2 t \rightarrow \frac{T}{c_1} = e^{-6\lambda^2 t} \Rightarrow \boxed{T = c_1\cdot e^{-6\lambda^2 t}}$$

$$X'' + \lambda^2 X = 0$$

$$k^2 + \lambda^2 = 0 \Rightarrow k^2 = -\lambda^2 \rightarrow k^2 = \lambda^2\cdot i^2 \rightarrow k = 0 \pm \lambda i$$

$$\rightarrow X = c_2\cos(\lambda x) + c_3\cdot\sin(\lambda x)$$

$$\Rightarrow \boxed{u(x,t) = X\cdot T = e^{-6\lambda^2 t}\left(A\cos(\lambda x) + B\sin(\lambda x)\right)}$$

$$\boxed{(A = c_1\cdot c_2\ ;\ B = c_1\cdot c_3)}$$

שאלה 2 סעיף ב

ב

התחלה $u(x,0) = -4\sin(\pi x) + 2\sin(6\pi x) - \sin(4\pi x)$

שפה $u(0,t) = u(1,t) = 0$

$$u(0,t) = \underbrace{e^{-6\lambda^2 t}}_{\text{לא מתאפס}}\cdot A\overbrace{\cos(0)}^{=1} = 0$$

$$\boxed{A = 0}$$

$$u(1,t) = \underbrace{e^{-6\lambda^2 t}}_{\text{לא מתאפס}}\cdot B\sin(\lambda) \neq 0$$

לא יכול להתאפס

$$\sin(\lambda) = 0$$

$$\boxed{\lambda = n\pi}$$

המשך שאלה מס' 2 סעיף ב'

$(n\pi)^2$

$$u(x,t) = e^{-6\pi^2 t} \cdot B\sin(n\pi x)$$

$$u(x,0) = B\sin(n\pi x) = -4\sin(\pi x) + 2\sin(6\pi x) - \sin(4\pi x)$$

$B_1 = -4 \; , \; n=1$

$B_2 = 2 \; , \; n=6$

$B_3 = -1 \; , \; n=4$

$$u(x,t) = \underbrace{e^{-6\pi^2 t} \cdot (-4)\sin(\pi x)}_{n=1} + \underbrace{e^{-216\pi^2 t} \cdot 2\sin(6\pi x)}_{n=6} - \underbrace{e^{-96\pi^2 t} \cdot \sin(4\pi x)}_{n=4}$$

שאלה 3 סעיף ג

$f(x) = -x \qquad 0 < x < 1$

$$b_n = \frac{2}{1}\int_0^1 -x \cdot \sin(n\pi x)\,dx$$

$v = -x \qquad v' = -1$

$u' = \sin(n\pi x) \qquad u = -\frac{\cos(n\pi x)}{n\pi}$

$$b_n = 2\left[\frac{x\cos(n\pi x)}{n\pi} - \int_0^1 \frac{\cos(n\pi x)}{n\pi}\,dx\right]$$

$$b_n = 2\left[\frac{x\cos(n\pi x)}{n\pi} - \frac{\sin(n\pi x)}{(n\pi)^2}\right]\Bigg|_0^1$$

$$b_n = 2\left[\left(\frac{\cos(n\pi)}{n\pi} - \underbrace{\cancel{\frac{\sin(n\pi)}{(n\pi)^2}}}_{\text{מתאפס}}\right) - (0-0)\right]$$

$$\Rightarrow b_n = \frac{2}{n\pi}\cdot\cos(n\pi) = \frac{2}{n\pi}\cdot(-1)^n$$

$n = 2k$

$n = 2k+1$

$$b_n = \begin{cases} \dfrac{2}{(2k)\pi} = \dfrac{1}{\pi \cdot k} & \text{זוגי} \\[2ex] \dfrac{-2}{(2k+1)\cdot\pi} & \text{אי-זוגי} \end{cases}$$

$$f(x) = \sum_{n=1}^{\infty} \underbrace{\frac{2(-1)^n}{n\pi}}_{b_n}\cdot\sin(n\pi x) \qquad //$$

שאלה 3 סעיף (ב)

3

ב

$$u(x,t) = e^{-6\lambda^2 t} \cdot B \sin(n\pi x)$$

המשך השאלה ב!

ראינו בשאלה $u(x,0) = -x$

$$u(x,t) = \sum_{n=1}^{\infty} e^{-6\lambda^2 t} \cdot \underbrace{\frac{2(-1)^n}{n\pi}}_{b_n \text{ שמצאנו}} \cdot \sin(n\pi x)$$

$$u(x,t) = \sum_{n=1}^{\infty} e^{-6(n\pi)^2 t} \cdot \frac{2(-1)^n}{n\pi} \cdot \sin(n\pi x) \quad //$$

$$u(x,t) = \underbrace{e^{-6\pi^2 t} \cdot \frac{-2}{\pi} \cdot \sin(\pi x)}_{n=1} + \underbrace{e^{-24\pi^2 t} \cdot \frac{1}{\pi} \cdot \sin(2\pi x)}_{n=2} \underbrace{- e^{-54\pi^2 t} \cdot \frac{2}{3\pi} \cdot \sin(3\pi x)}_{n=3} \ldots$$

פתרון

שאלה 5 סעיף א (1)

5

א

(1) $J_{n+1}(x) = \frac{2n}{x} \cdot J_n(x) - J_{n-1}(x)$

$n=2$

$J_{2+1}(x) = \frac{2\cdot 2}{x} \cdot J_2(x) - J_{2-1}(x) = \frac{4}{x} \cdot J_2 - J_1(x) = J_3(x)$

$n=1$

$J_2 = \frac{2}{x} \cdot J_1(x) - J_0(x)$

$J_3 = \frac{4}{x}\Big(\overbrace{\frac{2}{x} \cdot J_1(x) - J_0(x)}^{J_2(x)}\Big) - J_1(x)$

$J_3 = \frac{8}{x^2} \cdot J_1(x) - \frac{4}{x} \cdot J_0 - J_1(x)$

$\Rightarrow J_3(x) = J_1(x) \cdot \left[\frac{8}{x^2} - 1\right] - \frac{4}{x} J_0(x)$ //

שאלה 5 סעיף א (2)

(2)

$\int x^3 \cdot J_2 \, dx = \int x \cdot x^2 \cdot J_2 \, dx$

$v = x \quad v' = 1$

$u' = x^2 \cdot J_2 \quad u = x^2 \cdot J_3$

$= x^3 \cdot J_3 - \int x^2 \cdot J_3 \, dx \Rightarrow x^3 \cdot J_3 - x^2 \cdot J_4(x)$ //

שאלה 5 סעיף ב (1)

(2) לחשב את P_3 באמצעות רודריגס

(n=3) $$P_3(x) = \frac{1}{2^3 \cdot 3!}\frac{d^3}{dx^3}(x^2-1)^3$$

(1) $3(x^2-1)^2 \cdot 2x = 6x(x^2-1)^2$

(2) $6(x^2-1)^2 + 6x \cdot 2(x^2-1) \cdot 2x$

$6(x^2-1)^2 + \underbrace{24x^2(x^2-1)}_{24x^4-24x^2}$

(3) $\cancel{12(x^2-1)\cdot 2x + 48x(x^2-1) + 24x^2(}$

$24x(x^2-1) + 96x^3 - 48x$

$24x^3 - 24x + 96x^3 - 48x \rightarrow \underbrace{96x^3 + 24x^3}\underbrace{-24x - 48x}$

$120x^3 - 72x$

$$P_3(x) = \frac{1}{48}(120x^3 - 72x) = \frac{5}{2}x^3 - \frac{3}{2}x \quad //$$

שאלה 5 סעיף ב (2)

לחשב את המקדמים $x^4 - 3x^2 + x$ עבור $\sum_{k=0}^{\infty} A_k P_k(x)$

(2)

$$A_k = \frac{2}{J_4^2(\lambda_p)}\int_0^1 x \cdot J_3(\lambda_p x)\cdot(x^4-3x^2+x)\,dx = \int_0^1 J_3(\lambda_p x)(x^5-3x^3+x^2)\,dx$$

$\lambda_p \cdot x = v \rightarrow x = \frac{v}{\lambda_p} \rightarrow dx = \frac{1}{\lambda_p}dv$

הגבולות משתנים לגבולות של v: $0 < v < \lambda_p$

$$A_k = \frac{2}{J_4^2(\lambda_p)}\int_0^{\lambda_p} J_3(v)\cdot\frac{1}{\lambda_p}(v^5-3v^3+v^2)\frac{dv}{\lambda_p}$$

$$A_k = \frac{2}{J_4^2(\lambda_p)\cdot\lambda_p^2}\int_0^{\lambda_p} J_3(v)\cdot(v^5-3v^3+v^2)\,dv = \frac{2}{J_4^2(\lambda_p)\cdot\lambda_p^2}\left(v^5-3v^3+v^2\right)\cdot J_4(v)\Big|_0^{\lambda_p}$$

שאלה 5

המשך שאלה 5 סעיף ב 2

$$\frac{2}{J_4^2(\lambda_p)\cdot\lambda_p^2}\cdot\left(v^5-3v^3+v^2\right)\cdot J_4(v)\Big|_0^{\lambda_p}$$

$$\frac{2}{J_4^2(\lambda_p)\cdot\lambda_p^2}\left(\left((\lambda_p)^5-3(\lambda_p)^3+(\lambda_p)^2\right)\cdot J_4(\lambda_p)-0\right)=$$

$$\frac{2}{J_4^2(\lambda_p)}\left(\lambda_p^3-3\lambda_p+1\right)\cdot J_4(\lambda_p)=\frac{2}{J_4(\lambda_p)}\cdot\left(\lambda_p^3-3\lambda_p+1\right)=A_p \,//$$

$$f(x)=x^4-3x^2+x=\sum_{p=0}^{\infty}\frac{2}{J_4(\lambda_p)}\cdot\left(\lambda_p^3-3\lambda_p+1\right)$$